Correction contrôle continu du 13/03/2012

Ex. 1

1). $\displaystyle\int \frac{\ln(x+3)\,dx}{(x-3)^3} = \int \ln(x+3)\, d\left(-\frac{1}{2(x-3)^2}\right) = -\frac{\ln(x+3)}{2(x-3)^2} + \frac{1}{2}\int \frac{dx}{(x-3)^2(x+3)} \ominus$

$$\frac{1}{(x-3)^2(x+3)} = \frac{A}{(x-3)^2} + \frac{B}{x-3} + \frac{C}{x+3}$$

$$A = \lim_{x\to 3} \frac{1}{x+3} = \frac{1}{6}$$

$$C = \lim_{x\to -3} \frac{1}{(x-3)^2} = \frac{1}{36}$$

$$\frac{1}{6(x-3)^2} + \frac{B}{x-3} + \frac{1}{36(x+3)} = \frac{\left(B+\frac{1}{36}\right)x^2 + \dots x + \dots}{(x-3)^2(x+3)}$$

Donc $B = -\frac{1}{36}$

Par conséquent

$$\ominus\ -\frac{\ln(x+3)}{2(x-3)^2} + \frac{1}{2}\int\left(\frac{1}{6}\cdot\frac{1}{(x-3)^2} - \frac{1}{36(x-3)} + \frac{1}{36(x+3)}\right)dx =$$

$$= -\frac{\ln(x+3)}{2(x-3)^2} - \frac{1}{12(x-3)} + \frac{1}{72}\ln\frac{x+3}{x-3}.$$

2). $\displaystyle\int x^2\sin^3 x\,dx = \int x^2 \underbrace{\sin^2 x}_{\substack{\shortparallel \\ 1-\cos^2 x}}\, d(-\cos x) = \int x^2\, d\left(\frac{\cos^3 x}{3} - \cos x\right) =$

$$= x^2\left(\frac{\cos^3 x}{3} - \cos x\right) - 2\int x\left(\frac{\cos^3 x}{3} - \cos x\right)dx =$$

$$= x^2\left(\frac{\cos^3 x}{3} - \cos x\right) - 2\int x\left(\frac{\cos^2 x}{3} - 1\right)d(\sin x) =$$

$$= x^2\left(\frac{\cos^3 x}{3} - \cos x\right) + 2\int x\left(\frac{2}{3} + \frac{\sin^2 x}{3}\right)d(\sin x) =$$

$$= x^2\left(\frac{\cos^3 x}{3} - \cos x\right) + 2\int x\, d\left(\frac{2}{3}\sin x + \frac{\sin^3 x}{9}\right) =$$

$$= x^2\left(\frac{\cos^3 x}{3} - \cos x\right) + 2x\left(\frac{2}{3}\sin x + \frac{\sin^3 x}{9}\right) -$$

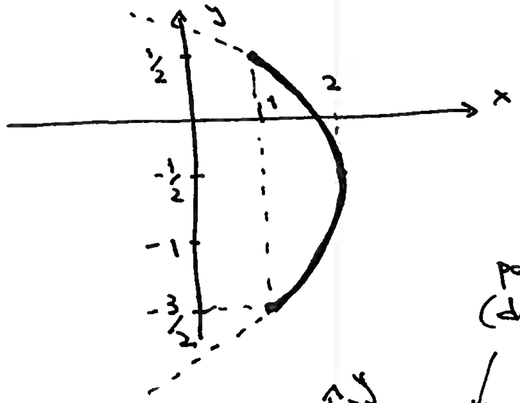
$$- \int\left(\frac{4}{3}\sin x + \frac{2}{9}\sin^3 x\right)dx =$$

$$= x^2\left(\frac{\cos^3 x}{3} - \cos x\right) + 2x\left(\frac{2}{3}\sin x + \frac{\sin^3 x}{9}\right) + \frac{4}{3}\cos x + \frac{2}{9}\int(1-\cos^2 x)\,d(\cos x)$$

$$= x^2\left(\frac{\cos^3 x}{3} - \cos x\right) + x\left(\frac{4}{3}\sin x + \frac{2}{9}\sin^3 x\right) + \left(\frac{14}{9}\cos x - \frac{2}{27}\cos^3 x\right).$$

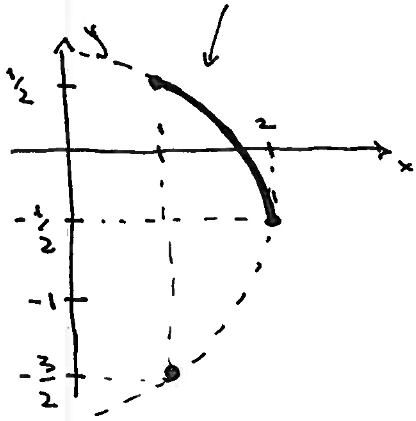
Ex.2)

1). $\begin{cases} x(t) = 2 - t^2 \\ y(t) = t - \frac{1}{2} \\ t \in [-1, 1] \end{cases} \Rightarrow \begin{array}{l} x = 2 - (y + \frac{1}{2})^2 \\ y \in [-\frac{3}{2}, \frac{1}{2}] \end{array}$

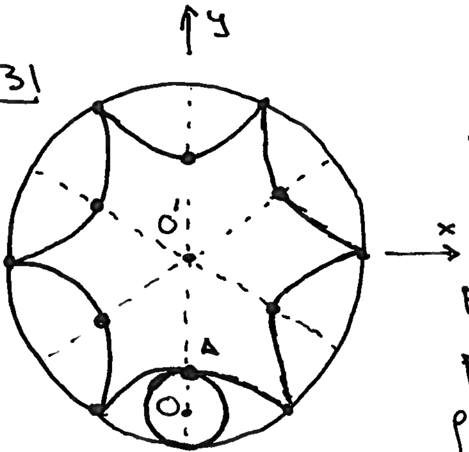


parcourue 2 fois (dans les 2 sens)

2). $\begin{cases} x(t) = 2 - t^4 \\ y(t) = t^2 - \frac{1}{2} \\ t \in [-1, 1] \end{cases} \Rightarrow \begin{array}{l} x = 2 - (y + \frac{1}{2})^2 \\ y \in [-\frac{1}{2}, \frac{1}{2}] \end{array}$



Ex.3)



- trajectoire fermée car après 6 tours (1 tour complet) le point revient à sa position initiale.

Equations paramétrique de la trajectoire pour le centre O

$$\begin{cases} x_O(t) = 5\cos(\Omega t - \pi/2) = 5 \sin \Omega t \\ y_O(t) = 5 \sin(\Omega t - \pi/2) = -5\cos \Omega t \end{cases}$$

Ici: Ω note la vitesse angulaire de rotation autour de O'.

De même, les équations paramétriques pour x_A, y_A par rapport à O :

$$\begin{cases} x_{A/O}(t) = 1 \cdot \cos(-\omega t + \frac{\pi}{2}) = \sin \omega t \\ y_{A/O}(t) = 1 \cdot \sin(-\omega t + \frac{\pi}{2}) = \cos \omega t \end{cases}$$

ω vitesse angulaire de rotation de A autour de O

Mais $\Omega = \dfrac{\omega}{5}$, donc on peut poser

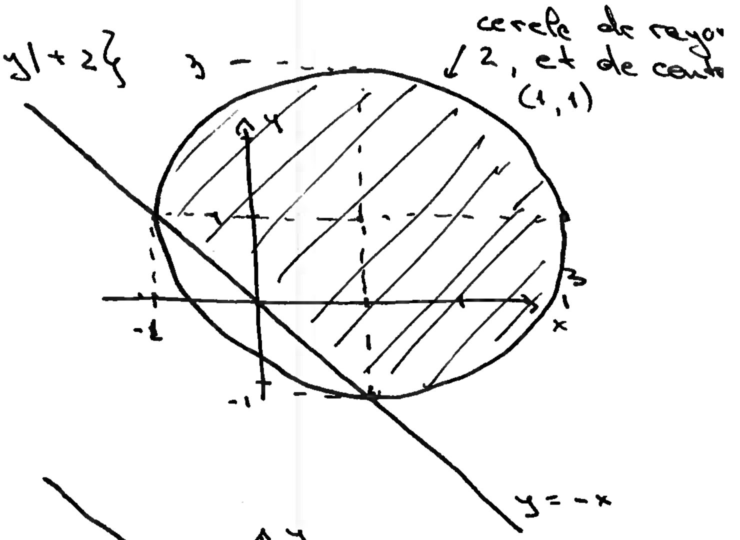
$$\begin{cases} x_A(t) = 5 \sin \Omega t + \sin 5\Omega t \\ y_A(t) = -5\cos \Omega t + \cos 5 \Omega t \end{cases}$$

Ex. 4 $D = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 2|x+y| + 2\}$

1). $x+y \geq 0 \iff y \geq -x$

$$x^2+y^2 \leq 2x+2y+2$$

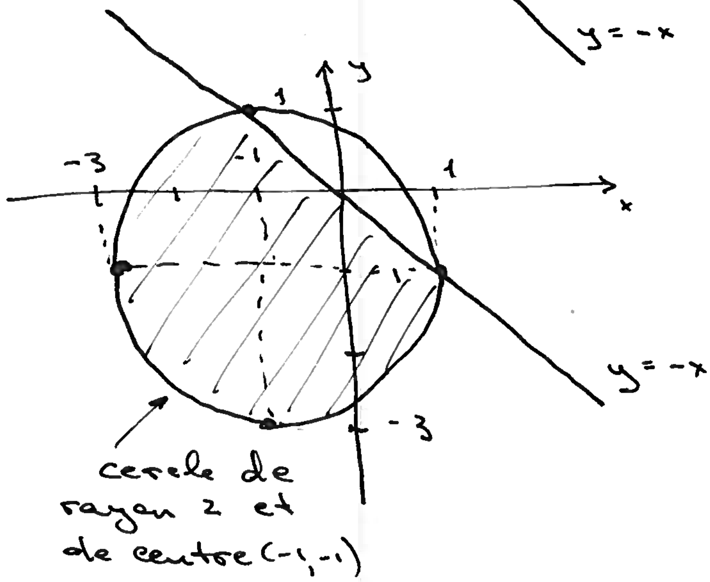
$$(x-1)^2+(y-1)^2 \leq 2^2$$



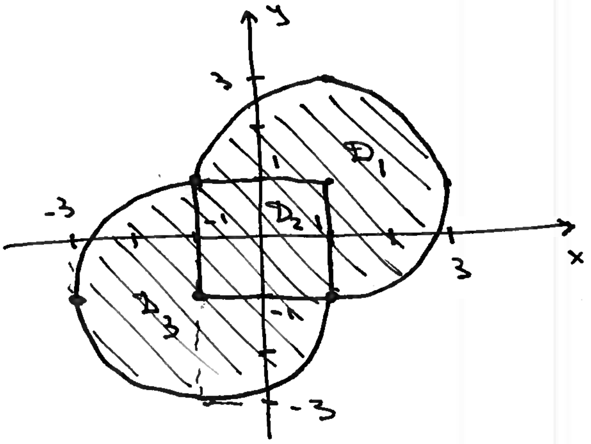
2). $x+y \leq 0 \iff y \leq -x$

$$x^2+y^2 \leq -2x-2y+2$$

$$(x+1)^2+(y+1)^2 \leq 2^2$$



Donc la forme du domaine :



$$\iint_D f\,dx\,dy = \iint_{D_1} + \iint_{D_2} + \iint_{D_3} =$$

$$= \int_0^2 \left(\int_{-\pi/2}^{\pi} f(1+r\cos\varphi, 1+r\sin\varphi)\,d\varphi \right) r\,dr$$

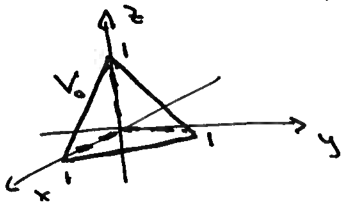
$$+ \int_{-1}^{1} \left(\int_{-1}^{1} f(x,y)\,dy \right) dx +$$

$$+ \int_0^2 \left(\int_{\pi/2}^{2\pi} f(-1+r\cos\varphi, -1+r\sin\varphi)\,d\varphi \right) r\,dr$$

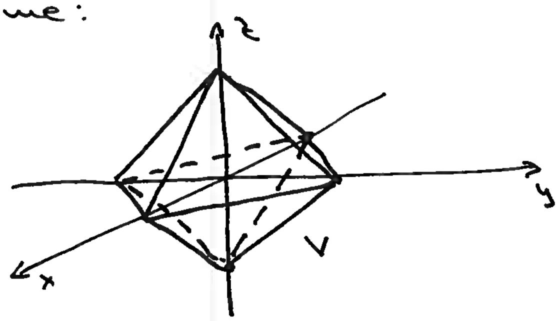
$$\text{aire}(D) = \text{aire}(D_1) + \text{aire}(D_2) + \text{aire}(D_3) =$$

$$= \frac{3}{4}\pi\, 2^2 + 2\cdot 2 + \frac{3}{4}\pi\cdot 2^2 = 4 + 6\pi$$

Ex.5 | Soient $x, y, z \geq 0$, alors $x+y+z \leq 1$ correspond à



Le domaine V est composé de 8 tetraedes de cette forme:



Comme $f(x,y,z) = |x|+|y|+|z|$ vérifie

$f(x,y,z) = f(-x,y,z) = f(x,-y,z) = f(x,y,-z)$

alors

$$\iiint_V (|x|+|y|+|z|)\,dx\,dy\,dz = 8 \iiint_{V_0} (|x|+|y|+|z|)\,dx\,dy\,dz =$$

$$= 8 \iiint_{V_0} (x+y+z)\,dx\,dy\,dz = 8 \int_0^1 \left(\int_0^{1-x} \left(\int_0^{1-x-y} (x+y+z)\,dz \right) dy \right) dx =$$

$$= 8 \int_0^1 \left(\int_0^{1-x} \left((x+y)(1-x-y) + \frac{(1-x-y)^2}{2} \right) dy \right) dx =$$

$$= 8 \int_0^1 \left(\int_0^{1-x} \frac{1}{2}\left[1-(x+y)^2\right] dy \right) dx = 4 \int_0^1 \left[y - \frac{(x+y)^3}{3} \right]_0^{1-x} dx =$$

$$= 4 \int_0^1 \left(\frac{2}{3} - x + \frac{x^3}{3} \right) dx = 4 \left(\frac{2}{3} - \frac{1}{2} + \frac{1}{12} \right) = 1.$$